Blitzer

COLLEGE ALGEBRA

7th Edition

A Brief Guide to **Getting the Most** from This Book

1 Read the Book

Feature	Description	Benefit
Section-Opening Scenarios	Every section opens with a scenario presenting a unique application of algebra in your life outside the classroom.	Realizing that algebra is everywhere will help motivate your learning. **(See page 106.)**
Detailed Worked-Out Examples	Examples are clearly written and provide step-by-step solutions. No steps are omitted, and each step is thoroughly explained to the right of the mathematics.	The blue annotations will help you understand the solutions by providing the reason why every algebraic step is true. **(See page 111.)**
Applications Using Real-World Data	Interesting applications from nearly every discipline, supported by up-to-date real-world data, are included in every section.	Ever wondered how you'll use algebra? This feature will show you how algebra can solve real problems. **(See page 265.)**
Great Question!	Answers to students' questions offer suggestions for problem solving, point out common errors to avoid, and provide informal hints and suggestions.	By seeing common mistakes, you'll be able to avoid them. This feature should help you not to feel anxious or threatened when asking questions in class. **(See page 109.)**
Brief Reviews	NEW to this edition. Brief Reviews cover skills you already learned but may have forgotten.	Having these refresher boxes easily accessible will help ease anxiety about skills you may have forgotten. **(See page 478.)**
Achieving Success	NEW to this edition. Achieving Success boxes offer strategies for persistence and success in college mathematics courses.	Follow these suggestions to help achieve your full academic potential in college mathematics. **(See page 166.)**
Explanatory Voice Balloons	Voice balloons help to demystify algebra. They translate mathematical language into plain English, clarify problem-solving procedures, and present alternative ways of understanding.	Does math ever look foreign to you? This feature often translates math into everyday English. **(See page 201.)**
Learning Objectives	Every section begins with a list of objectives. Each objective is restated in the margin where the objective is covered.	The objectives focus your reading by emphasizing what is most important and where to find it. **(See page 124.)**
Technology	The screens displayed in the technology boxes show how graphing utilities verify and visualize algebraic results.	Even if you are not using a graphing utility in the course, this feature will help you understand different approaches to problem solving. **(See page 110.)**

2 Work the Problems

Feature	Description	Benefit
Check Point Examples	Each example is followed by a matched problem, called a Check Point, that offers you the opportunity to work a similar exercise. The answers to the Check Points are provided in the answer section.	You learn best by doing. You'll solidify your understanding of worked examples if you try a similar problem right away to be sure you understand what you've just read. **(See page 288.)**
Concept and Vocabulary Checks	These short-answer questions, mainly fill-in-the-blank and true/false items, assess your understanding of the definitions and concepts presented in each section.	It is difficult to learn algebra without knowing its special language. These exercises test your understanding of the vocabulary and concepts. **(See page 229.)**
Extensive and Varied Exercise Sets	An abundant collection of exercises is included in an Exercise Set at the end of each section. Exercises are organized within several categories. Your instructor will usually provide guidance on which exercises to work. The exercises in the first category, Practice Exercises, follow the same order as the section's worked examples.	The parallel order of the Practice Exercises lets you refer to the worked examples and use them as models for solving these problems. **(See page 406.)**
Practice Plus Problems	This category of exercises contains more challenging problems that often require you to combine several skills or concepts.	It is important to dig in and develop your problem-solving skills. Practice Plus Exercises provide you with ample opportunity to do so. **(See page 407.)**
Retaining the Concepts	NEW to this edition. Beginning with Chapter 2, each Exercise Set contains review exercises under the header "Retaining the Concepts."	These exercises improve your understanding of the topics and help maintain mastery of the material. **(See page 234.)**
Preview Problems	Each Exercise Set concludes with three problems to help you prepare for the next section.	These exercises let you review previously covered material that you'll need to be successful for the forthcoming section. Some of these problems will get you thinking about concepts you'll soon encounter. **(See page 312.)**

3 Review for Quizzes and Tests

Feature	Description	Benefit
Mid-Chapter Check Points	At approximately the midway point in the chapter, an integrated set of review exercises allows you to review the skills and concepts you learned separately over several sections.	By combining exercises from the first half of the chapter, the Mid-Chapter Check Points give a comprehensive review before you move on to the material in the remainder of the chapter. **(See page 281.)**
Chapter Review Grids	Each chapter contains a review chart that summarizes the definitions and concepts in every section of the chapter. Examples that illustrate these key concepts are also referenced in the chart.	Review this chart and you'll know the most important material in the chapter! **(See page 454.)**
Chapter Review Exercises	A comprehensive collection of review exercises for each of the chapter's sections follows the grid.	Practice makes perfect. These exercises contain the most significant problems for each of the chapter's sections. **(See page 209.)**
Chapter Tests	Each chapter contains a practice test with approximately 25 problems that cover the important concepts in the chapter. Take the practice test, check your answers, and then watch the Chapter Test Prep Videos to see worked-out solutions for any exercises you miss.	You can use the chapter test to determine whether you have mastered the material covered in the chapter. **(See page 213.)**
Chapter Test Prep Videos	These videos contain worked-out solutions to every exercise in each chapter test and can be found in MyMathLab and on YouTube.	The videos let you review any exercises you miss on the chapter test.
Objective Videos	NEW to this edition. These fresh, interactive videos walk you through the concepts from every objective of the text.	The videos provide you with active learning at your own pace.
Cumulative Review Exercises	Beginning with Chapter 2, each chapter concludes with a comprehensive collection of mixed cumulative review exercises. These exercises combine problems from previous chapters and the present chapter, providing an ongoing cumulative review.	Ever forget what you've learned? These exercises ensure that you are not forgetting anything as you move forward. **(See page 461.)**

COLLEGE ALGEBRA

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COLLEGE ALGEBRA

7th EDITION

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Pearson

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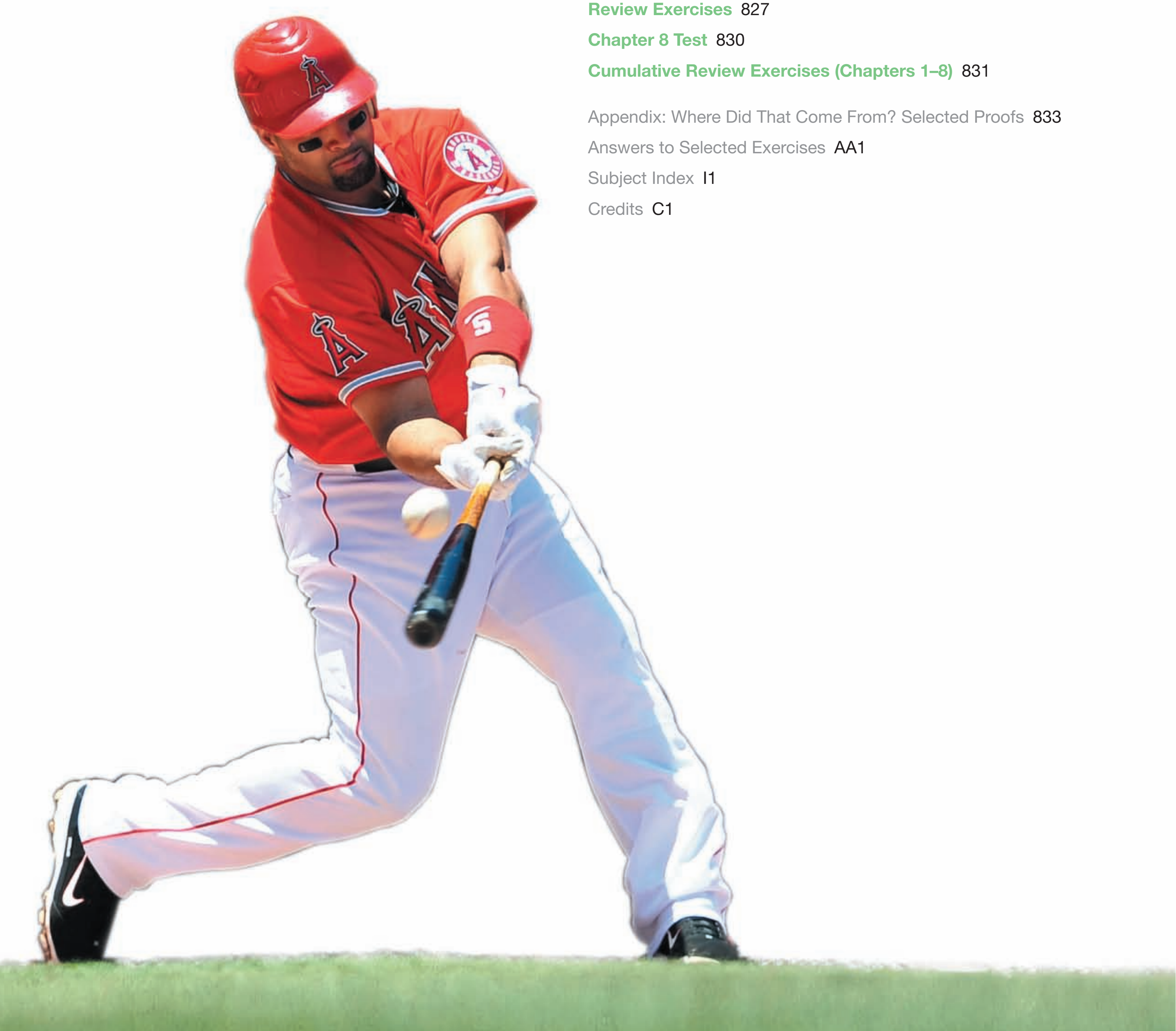

PREFACE

I've written ***College Algebra*, Seventh Edition**, to help diverse students, with different backgrounds and future goals, to succeed. The book has three fundamental goals:

1. To help students acquire a solid foundation in algebra, preparing them for other courses such as calculus, business calculus, and finite mathematics.
2. To show students how algebra can model and solve authentic real-world problems.
3. To enable students to develop problem-solving skills, while fostering critical thinking, within an interesting setting.

One major obstacle in the way of achieving these goals is the fact that very few students actually read their textbook. This has been a regular source of frustration for me and for my colleagues in the classroom. Anecdotal evidence gathered over years highlights two basic reasons that students do not take advantage of their textbook:

- "I'll never use this information."
- "I can't follow the explanations."

I've written every page of the Seventh Edition with the intent of eliminating these two objections. The ideas and tools I've used to do so are described for the student in "A Brief Guide to Getting the Most from This Book," which appears at the front of the book.

What's New in the Seventh Edition?

- **New Applications and Real-World Data.** The Seventh Edition contains 63 worked-out examples and exercises based on new data sets, and 36 examples and exercises based on data updated from the Sixth Edition. Many of the new applications involve topics relevant to college students, including student-loan debt (Chapter P, Mid-Chapter Check Point, Exercise 31), grade inflation (Exercise Set 1.2, Exercises 97–98), median earnings, by final degree earned (Exercise Set 1.3, Exercises 3–4), excuses for not meeting deadlines (Chapter 1 Summary, Exercise 36), political orientation of college freshmen (Chapter 2 Summary, Exercise 53), sleep hours of college students (Exercise Set 5.1, Exercise 74), and the number of hours college students study per week, by major (Exercise Set 5.2, Exercises 33–34).

- **Brief Reviews.** Beginning with Chapter 1, the Brief Review boxes that appear throughout the book summarize mathematical skills, many of which are course prerequisites, that students have learned, but which many students need to review. This feature appears whenever a particular skill is first needed and eliminates the need for you to reteach that skill. For more detail, students are referred to the appropriate section and objective in a previous chapter where the topic is fully developed.

- **Achieving Success.** The Achieving Success boxes, appearing at the end of many sections in Chapters 1 through 5, offer strategies for persistence and success in college mathematics courses.

- **Retaining the Concepts.** Beginning with Chapter 2, Section 2.1, each Exercise Set contains three review exercises under the header "Retaining the Concepts." These exercises are intended for students to review previously covered objectives in order to improve their understanding of the topics and to help maintain their mastery of the material. If students are not certain how to solve a review exercise, they can turn to the section and worked example given in parentheses at the end of each exercise. The Seventh Edition contains 78 new exercises in the "Retaining the Concepts" category.

- **New Blitzer Bonus Videos with Assessment.** Many of the Blitzer Bonus features throughout the textbook have been turned into animated videos that are built into the MyMathLab course. These videos help students make visual connections to algebra and the world around them. Assignable exercises have been created within the MyMathLab course to assess conceptual understanding and mastery. These videos and exercises can be turned into a media assignment within the Blitzer MyMathLab course.

- **Updated Learning Guide.** Organized by the textbook's learning objectives, this updated Learning Guide helps students learn how to make the most of their textbook for test preparation. Projects are now included to give students an opportunity to discover and reinforce the concepts in an active learning environment and are ideal for group work in class.

- **Updated Graphing Calculator Screens.** All screens have been updated using the TI-84 Plus C.

What Content and Organizational Changes Have Been Made to the Seventh Edition?

- **Section P.1 (Algebraic Expressions, Mathematical Models, and Real Numbers)** follows an example on the cost of attending college (Example 2) with a new Blitzer Bonus, "Is College Worthwhile?"

- **Section P.6 (Rational Expressions)** uses the least common denominator to combine rational expressions with different denominators, including expressions having no common factors in their denominators.

- **Section 1.1 (Graphing and Graphing Utilities)** contains a new example of a graph with more than one x-intercept (Example 5(d)).

- **Section 1.4 (Complex Numbers)** includes a new example on dividing complex numbers where the numerator is of the form bi (Example 3). (This is then followed by an example picked up from the Sixth Edition where the numerator is of the form $a + bi$.)
- **Section 1.5 (Quadratic Equations)** provides a step-by-step procedure for solving quadratic equations by completing the square. This procedure forms the framework for the solutions in Examples 4 and 5.
- **Section 1.5 (Quadratic Equations)** contains an example on the quadratic formula (Example 6) where the formula is used to solve a quadratic equation with rational solutions, an equation that students can also solve by factoring.
- **Section 1.5 (Quadratic Equations)** has a new application of the Pythagorean Theorem (Example 11) involving HDTV screens. The example is followed by a new Blitzer Bonus, "Screen Math."
- **Section 1.6 (Other Types of Equations)** includes an example on solving an equation quadratic in form (Example 8),

$$(x^2 - 5)^2 + 3(x^2 - 5) - 10 = 0,$$

where u is a binomial ($u = x^2 - 5$).
- **Section 2.2 (More on Functions and Their Graphs)** contains a new discussion on graphs with three forms of symmetry (Examples 2 and 3) before presenting even and odd functions. A new example (Example 4) addresses identifying even or odd functions from graphs.
- **Section 2.3 (Linear Functions and Slope)** includes a new Blitzer Bonus, "Slope and Applauding Together."
- **Section 2.7 (Inverse Functions)** replaces an example on finding the inverse of $f(x) = \frac{5}{x} + 4$ with an example on finding the inverse of $f(x) = \frac{x + 2}{x - 3}$ (Example 4), a function with two occurrences of x.
- **Section 3.5 (Rational Functions and Their Graphs)** opens with a discussion of college students and video games. This is revisited in a new example (Example 9, "Putting the Video-Game Player Inside the Game") involving the Oculus Rift, a virtual reality headset that enables users to experience video games as immersive three-dimensional environments.
- **Section 5.1 (Systems of Linear Equations in Two Variables)** contains a new discussion on problems involving mixtures, important for many STEM students. A new example (Example 8) illustrates the procedure for solving a mixture problem.
- **Section 6.1 (Matrix Solutions to Linear Systems)** has a new opening example (Example 1) showing the details on how to write an augmented matrix.
- **Section 7.1 (The Ellipse)** includes a new example (Example 5) showing the details on graphing an ellipse centered at (h, k) by completing the square.
- **Section 7.3 (The Parabola)** adds a new objective on identifying conics of the form $Ax^2 + Cy^2 + Dx + Ey + F = 0$ without completing the square, supported by an example (Example 7).
- **Section 8.2 (Arithmetic Sequences)** contains a new example (Example 3) on writing the general term of an arithmetic sequence.
- **Section 8.7 (Probability)** uses the popular lottery games Powerball (Example 5) and Mega Millions (Exercises 27–30) as applications of probability and combinations.

What Familiar Features Have Been Retained in the Seventh Edition?

- **Learning Objectives.** Learning objectives, framed in the context of a student question (What am I supposed to learn?), are clearly stated at the beginning of each section. These objectives help students recognize and focus on the section's most important ideas. The objectives are restated in the margin at their point of use.
- **Chapter-Opening and Section-Opening Scenarios.** Every chapter and every section open with a scenario presenting a unique application of mathematics in students' lives outside the classroom. These scenarios are revisited in the course of the chapter or section in an example, discussion, or exercise.
- **Innovative Applications.** A wide variety of interesting applications, supported by up-to-date, real-world data, are included in every section.
- **Detailed Worked-Out Examples.** Each example is titled, making the purpose of the example clear. Examples are clearly written and provide students with detailed step-by-step solutions. No steps are omitted and each step is thoroughly explained to the right of the mathematics.
- **Explanatory Voice Balloons.** Voice balloons are used in a variety of ways to demystify mathematics. They translate algebraic ideas into everyday English, help clarify problem-solving procedures, present alternative ways of understanding concepts, and connect problem solving to concepts students have already learned.
- **Check Point Examples.** Each example is followed by a similar matched problem, called a Check Point, offering students the opportunity to test their understanding of the example by working a similar exercise. The answers to the Check Points are provided in the answer section.

- **Concept and Vocabulary Checks.** This feature offers short-answer exercises, mainly fill-in-the-blank and true/false items, that assess students' understanding of the definitions and concepts presented in each section. The Concept and Vocabulary Checks appear as separate features preceding the Exercise Sets.

- **Extensive and Varied Exercise Sets.** An abundant collection of exercises is included in an Exercise Set at the end of each section. Exercises are organized within nine category types: Practice Exercises, Practice Plus Exercises, Application Exercises, Explaining the Concepts, Technology Exercises, Critical Thinking Exercises, Group Exercises, Retaining the Concepts, and Preview Exercises. This format makes it easy to create well-rounded homework assignments. The order of the Practice Exercises is exactly the same as the order of the section's worked examples. This parallel order enables students to refer to the titled examples and their detailed explanations to achieve success working the Practice Exercises.

- **Practice Plus Problems.** This category of exercises contains more challenging practice problems that often require students to combine several skills or concepts. With an average of ten Practice Plus problems per Exercise Set, instructors are provided with the option of creating assignments that take Practice Exercises to a more challenging level.

- **Mid-Chapter Check Points.** At approximately the midway point in each chapter, an integrated set of Review Exercises allows students to review and assimilate the skills and concepts they learned separately over several sections.

- **Graphing and Functions.** Graphing is introduced in Chapter 1 and functions are introduced in Chapter 2, with an integrated graphing functional approach emphasized throughout the book. Graphs and functions that model data appear in nearly every section and Exercise Set. Examples and exercises use graphs of functions to explore relationships between data and to provide ways of visualizing a problem's solution. Because functions are the core of this course, students are repeatedly shown how functions relate to equations and graphs.

- **Integration of Technology Using Graphic and Numerical Approaches to Problems.** Side-by-side features in the technology boxes connect algebraic solutions to graphic and numerical approaches to problems. Although the use of graphing utilities is optional, students can use the explanatory voice balloons to understand different approaches to problems even if they are not using a graphing utility in the course.

- **Great Question!** This feature presents a variety of study tips in the context of students' questions. Answers to questions offer suggestions for problem solving, point out common errors to avoid, and provide informal hints and suggestions. As a secondary benefit, this feature should help students not to feel anxious or threatened when asking questions in class.

- **Chapter Summaries.** Each chapter contains a review chart that summarizes the definitions and concepts in every section of the chapter. Examples that illustrate these key concepts are also referenced in the chart.

- **End-of-Chapter Materials.** A comprehensive collection of Review Exercises for each of the chapter's sections follows the Summary. This is followed by a Chapter Test that enables students to test their understanding of the material covered in the chapter. Beginning with Chapter 2, each chapter concludes with a comprehensive collection of mixed Cumulative Review Exercises.

- **Blitzer Bonuses.** These enrichment essays provide historical, interdisciplinary, and otherwise interesting connections to the algebra under study, showing students that math is an interesting and dynamic discipline.

- **Discovery.** Discovery boxes, found throughout the text, encourage students to further explore algebraic concepts. These explorations are optional and their omission does not interfere with the continuity of the topic under consideration.

I hope that my passion for teaching, as well as my respect for the diversity of students I have taught and learned from over the years, is apparent throughout this new edition. By connecting algebra to the whole spectrum of learning, it is my intent to show students that their world is profoundly mathematical, and indeed, π is in the sky.

Robert Blitzer

Acknowledgments

An enormous benefit of authoring a successful series is the broad-based feedback I receive from the students, dedicated users, and reviewers. Every change to this edition is the result of their thoughtful comments and suggestions. I would like to express my appreciation to all the reviewers, whose collective insights form the backbone of this revision. In particular, I would like to thank the following people for reviewing *College Algebra, Algebra and Trigonometry, Precalculus*, and *Trigonometry*.

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Robert Blitzer

Get the Most Out of MyMathLab®

MyMathLab is the leading online homework, tutorial, and assessment program for teaching and learning mathematics, built around Pearson's best-selling content. MyMathLab helps students and instructors improve results; it provides engaging experiences and personalized learning for each student so learning can happen in any environment. Plus, it offers flexible and time-saving course management features to allow instructors to easily manage their classes while remaining in complete control, regardless of course format.

Preparedness

MyMathLab course solutions offer a complete College Algebra or Precalculus course with integrated review of select topics from developmental algebra. These courses help remediate students "just-in-time" and help with student retention of important concepts, ultimately boosting student success.

- Students begin each chapter by completing a Skills Check assignment to pinpoint which developmental topics, if any, they need to review.
- Students who demonstrate mastery of the review topics will move straight into the College Algebra content.
- A personalized review homework assignment will provide extra support for the students who need it.
- Additional review materials (worksheets, videos, and more) are available in an Integrated Review section at the start of each chapter in MyMathLab.

MyMathLab with Integrated Review are appropriate for students who struggle with pre-requisite skills and for co-requisite course models. These Integrated Review MyMathLab courses are available for a variety of College Algebra and Precalculus programs, as well as a variety of other disciplines.

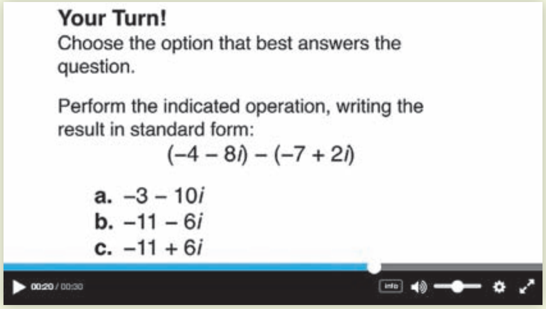
Used by more than 37 million students worldwide, MyMathLab delivers consistent, measurable gains in student learning outcomes, retention, and subsequent course success.

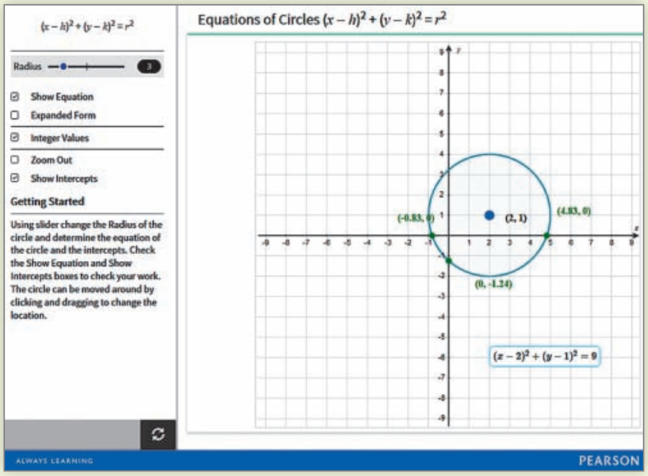
MyMathLab Online Course for *College Algebra* by Robert Blitzer

(access code required)

NEW! Video Program

These fresh, interactive videos walk you through the concepts from every objective of the text. The videos provide an active learning environment where students can work at their own pace.





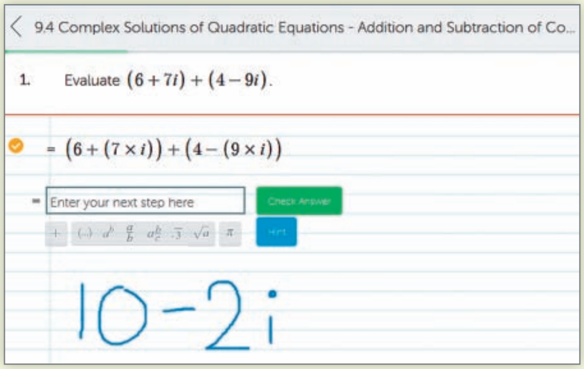
NEW! Guided Visualizations

These HTML-based, interactive figures help students visualize the concepts through directed explorations and purposeful manipulation. They encourage active learning, critical thinking, and conceptual learning. They are compatible with iPad and tablet devices.

The Guided Visualizations are located in the Multimedia Library and can be assigned as homework with correlating assessment exercises. Additional Exploratory Exercises are available to help students think more conceptually about the figures and provide an excellent framework for group projects or lecture discussion.

NEW! Workspace Assignments

Students can now show their work like never before! Workspace Assignments allow students to work through an exercise step-by-step, and show their mathematical reasoning as they progress. Students receive immediate feedback after they complete each step, and helpful hints and videos offer guidance when they need it. When accessed via a mobile device, Workspace exercises use handwriting recognition software that allows students to naturally write out their answers. Each student's work is automatically graded and captured in the MyMathLab gradebook so instructors can easily pinpoint exactly where they need to focus their instruction.



www.mymathlab.com

Resources for Success

Instructor Resources

Additional resources can be downloaded from **www.mymathlab.com** or **www.pearsonhighered.com** or hardcopy resources can be ordered from your sales representative.

Annotated Instructor's Edition

Shorter answers are on the page beside the exercises. Longer answers are in the back of the text.

Instructor's Solutions Manual

Fully worked solutions to all textbook exercises.

PowerPoint® Lecture Slides

Fully editable lecture slides that correlate to the textbook.

Mini Lecture Notes

Additional examples and helpful teaching tips for each section.

TestGen®

Enables instructors to build, edit, print, and administer tests using a computerized bank of algorithmic questions developed to cover all the objectives of the text.

Student Resources

Additional resources to help student success are available to be packaged with the Blitzer textbook and MyMathLab access code.

Objective Level Videos

These fresh, interactive videos walk students through the concepts from every objective of the text. The videos provide an active learning environment where students can work at their own pace.

Chapter Test Prep Videos

Students can watch instructors work through step-by-step solutions to all the Chapter Test exercises from the textbook. These are available in MyMathLab and on YouTube.

Student Solutions Manual

Fully worked solutions to odd-numbered exercises and available to be packaged with the textbook.

Learning Guide

This note-taking guide is organized by objective and begins each chapter with an engaging application, providing additional examples and exercises for students to work through for a greater conceptual understanding and mastery of mathematical topics. New to this edition: classroom projects are included with each chapter providing students the opportunity to work collaboratively for stronger conceptual understanding. The Learning Guide is available in PDF and customizable Word file formats in MyMathLab. It can also be packaged with the textbook and MyMathLab access code.

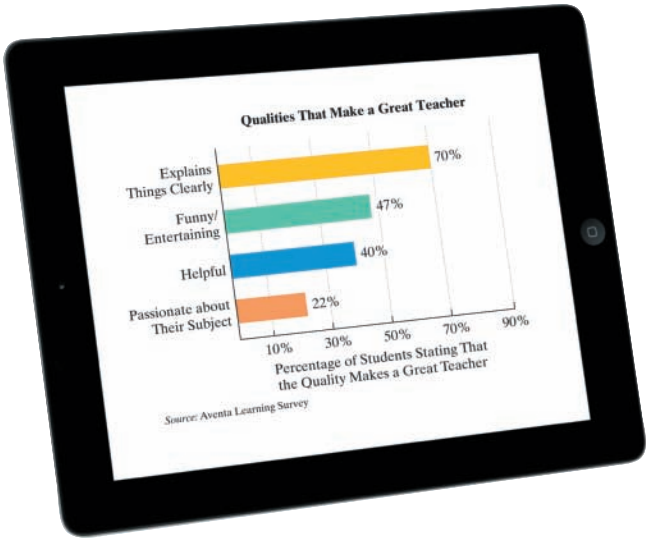
MathTalk Videos

Engaging videos connect mathematics to real-life events and interesting applications. These fun, instructional videos show students that math is relevant to their daily lives and are assignable in MyMathLab. Assignable exercises are available in MyMathLab for these videos to help students retain valuable information presented in the videos.

www.mymathlab.com

TO THE STUDENT

The bar graph shows some of the qualities that students say make a great teacher. It was my goal to incorporate each of these qualities throughout the pages of this book.



Explains Things Clearly

I understand that your primary purpose in reading *College Algebra* is to acquire a solid understanding of the required topics in your algebra course. In order to achieve this goal, I've carefully explained each topic. Important definitions and procedures are set off in boxes, and worked-out examples that present solutions in a step-by-step manner appear in every section. Each example is followed by a similar matched problem, called a Check Point, for you to try so that you can actively participate in the learning process as you read the book. (Answers to all Check Points appear in the back of the book.)

Funny & Entertaining

Who says that an algebra textbook can't be entertaining? From our unusual cover to the photos in the chapter and section openers, prepare to expect the unexpected. I hope some of the book's enrichment essays, called Blitzer Bonuses, will put a smile on your face from time to time.

Helpful

I designed the book's features to help you acquire knowledge of college algebra, as well as to show you how algebra can solve authentic problems that apply to your life. These helpful features include:

- **Explanatory Voice Balloons:** Voice balloons are used in a variety of ways to make math less intimidating. They translate algebraic language into everyday English, help clarify problem-solving procedures, present alternative ways of understanding concepts, and connect new concepts to concepts you have already learned.
- **Great Question!:** The book's Great Question! boxes are based on questions students ask in class. The answers to these questions give suggestions for problem solving, point out common errors to avoid, and provide informal hints and suggestions.
- **Achieving Success:** The book's Achieving Success boxes give you helpful strategies for success in learning algebra, as well as suggestions that can be applied for achieving your full academic potential in future college coursework.
- **Chapter Summaries:** Each chapter contains a review chart that summarizes the definitions and concepts in every section of the chapter. Examples from the chapter that illustrate these key concepts are also referenced in the chart. Review these summaries and you'll know the most important material in the chapter!

Passionate about the Subject

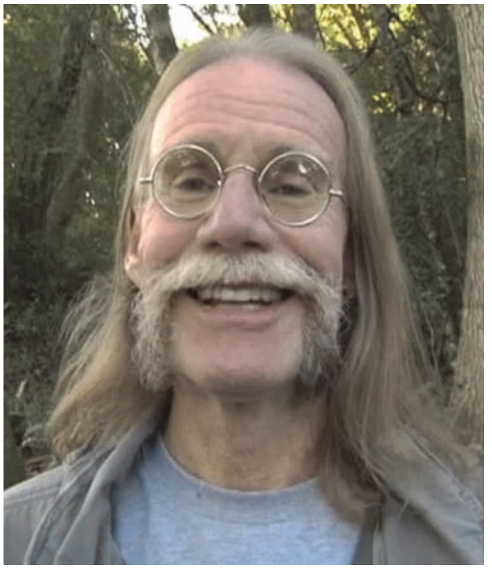
I passionately believe that no other discipline comes close to math in offering a more extensive set of tools for application and development of your mind. I wrote the book in Point Reyes National Seashore, 40 miles north of San Francisco. The park consists of 75,000 acres with miles of pristine surf-washed beaches, forested ridges, and bays bordered by white cliffs. It was my hope to convey the beauty and excitement of mathematics using nature's unspoiled beauty as a source of inspiration and creativity. Enjoy the pages that follow as you empower yourself with the algebra needed to succeed in college, your career, and your life.

Regards,

Bob

Robert Blitzer

ABOUT THE AUTHOR



Bob Blitzer is a native of Manhattan and received a Bachelor of Arts degree with dual majors in mathematics and psychology (minor: English literature) from the City College of New York. His unusual combination of academic interests led him toward a Master of Arts in mathematics from the University of Miami and a doctorate in behavioral sciences from Nova University. Bob's love for teaching mathematics was nourished for nearly 30 years at Miami Dade College, where he received numerous teaching awards, including Innovator of the Year from the League for Innovations in the Community College and an endowed chair based on excellence in the classroom. In addition to *College Algebra*, Bob has written textbooks covering developmental mathematics, introductory algebra, intermediate algebra, trigonometry, algebra and trigonometry, precalculus, and liberal arts mathematics, all published by Pearson. When not secluded in his Northern California writer's cabin, Bob can be found hiking the beaches and trails of Point Reyes National Seashore and tending to the chores required by his beloved entourage of horses, chickens, and irritable roosters.

APPLICATIONS INDEX

Prerequisites:
Fundamental Concepts of Algebra

CHAPTER
P

What can algebra possibly have to tell me about

- the skyrocketing cost of a college education?
- student-loan debt?
- my workouts?
- the effects of alcohol?
- the meaning of the national debt that is nearly $19 trillion?
- time dilation on a futuristic high-speed journey to a nearby star?
- racial bias?
- ethnic diversity in the United States?
- the widening imbalance between numbers of women and men on college campuses?

This chapter reviews fundamental concepts of algebra that are prerequisites for the study of college algebra. Throughout the chapter, you will see how the special language of algebra describes your world.

HERE'S WHERE YOU'LL FIND THESE APPLICATIONS:

College costs: Section P.1, Example 2; Exercise Set P.1, Exercises 131–132

Student-loan debt: Mid-Chapter Check Point, Exercise 31

Workouts: Exercise Set P.1, Exercises 129–130

The effects of alcohol: Blitzer Bonus beginning on page 15

The national debt: Section P.2, Example 12

Time dilation: Blitzer Bonus on page 47

Racial bias: Exercise Set P.4, Exercises 91–92

U.S. ethnic diversity: Chapter P Review, Exercise 23

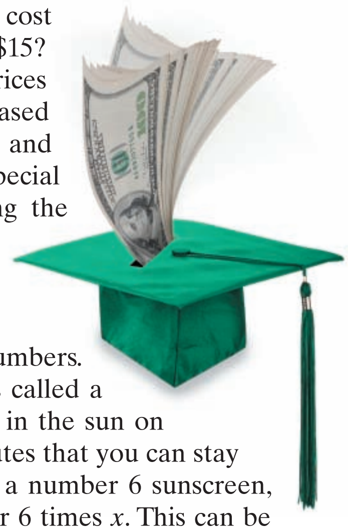
College gender imbalance: Chapter P Test, Exercise 32.

Section P.1 Algebraic Expressions, Mathematical Models, and Real Numbers

What am I supposed to learn?

After studying this section, you should be able to:

1. Evaluate algebraic expressions.
2. Use mathematical models.
3. Find the intersection of two sets.
4. Find the union of two sets.
5. Recognize subsets of the real numbers.
6. Use inequality symbols.
7. Evaluate absolute value.
8. Use absolute value to express distance.
9. Identify properties of the real numbers.
10. Simplify algebraic expressions.



How would your lifestyle change if a gallon of gas cost \$9.15? Or if the price of a staple such as milk was \$15? That's how much those products would cost if their prices had increased at the same rate college tuition has increased since 1980. (*Source*: Center for College Affordability and Productivity) In this section, you will learn how the special language of algebra describes your world, including the skyrocketing cost of a college education.

Algebraic Expressions

Algebra uses letters, such as x and y, to represent numbers. If a letter is used to represent various numbers, it is called a **variable**. For example, imagine that you are basking in the sun on the beach. We can let x represent the number of minutes that you can stay in the sun without burning with no sunscreen. With a number 6 sunscreen, exposure time without burning is six times as long, or 6 times x. This can be written $6 \cdot x$, but it is usually expressed as $6x$. Placing a number and a letter next to one another indicates multiplication.

Notice that $6x$ combines the number 6 and the variable x using the operation of multiplication. A combination of variables and numbers using the operations of addition, subtraction, multiplication, or division, as well as powers or roots, is called an **algebraic expression**. Here are some examples of algebraic expressions:

$$x + 6, \quad x - 6, \quad 6x, \quad \frac{x}{6}, \quad 3x + 5, \quad x^2 - 3, \quad \sqrt{x} + 7.$$

Many algebraic expressions involve *exponents*. For example, the algebraic expression

$$4x^2 + 330x + 3310$$

approximates the average cost of tuition and fees at public U.S. colleges for the school year ending x years after 2000. The expression x^2 means $x \cdot x$ and is read "x to the second power" or "x squared." The exponent, 2, indicates that the base, x, appears as a factor two times.

Exponential Notation

If n is a counting number (1, 2, 3, and so on),

Exponent or Power

$$b^n = \underbrace{b \cdot b \cdot b \cdot \cdots \cdot b}_{b \text{ appears as a factor } n \text{ times.}}.$$

Base

b^n is read "the nth power of b" or "b to the nth power." Thus, the nth power of b is defined as the product of n factors of b. The expression b^n is called an **exponential expression**. Furthermore, $b^1 = b$.

For example,

$$8^2 = 8 \cdot 8 = 64, \quad 5^3 = 5 \cdot 5 \cdot 5 = 125, \quad \text{and} \quad 2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16.$$

1 Evaluate algebraic expressions.

Evaluating Algebraic Expressions

Evaluating an algebraic expression means to find the value of the expression for a given value of the variable.

Many algebraic expressions involve more than one operation. Evaluating an algebraic expression without a calculator involves carefully applying the following order of operations agreement:

The Order of Operations Agreement

1. Perform operations within the innermost parentheses and work outward. If the algebraic expression involves a fraction, treat the numerator and the denominator as if they were each enclosed in parentheses.
2. Evaluate all exponential expressions.
3. Perform multiplications and divisions **as they occur**, working **from left to right**.
4. Perform additions and subtractions **as they occur**, working **from left to right**.

EXAMPLE 1 Evaluating an Algebraic Expression

Evaluate $7 + 5(x - 4)^3$ for $x = 6$.

SOLUTION

$$
\begin{aligned}
7 + 5(x - 4)^3 &= 7 + 5(6 - 4)^3 && \text{Replace } x \text{ with 6.} \\
&= 7 + 5(2)^3 && \text{First work inside parentheses: } 6 - 4 = 2. \\
&= 7 + 5(8) && \text{Evaluate the exponential expression: } 2^3 = 2 \cdot 2 \cdot 2 = 8. \\
&= 7 + 40 && \text{Multiply: } 5(8) = 40. \\
&= 47 && \text{Add.}
\end{aligned}
$$

Check Point 1 Evaluate $8 + 6(x - 3)^2$ for $x = 13$.

2 Use mathematical models.

Formulas and Mathematical Models

An **equation** is formed when an equal sign is placed between two algebraic expressions. One aim of algebra is to provide a compact, symbolic description of the world. These descriptions involve the use of *formulas*. A **formula** is an equation that uses variables to express a relationship between two or more quantities.

Here are two examples of formulas related to heart rate and exercise.

Couch-Potato Exercise

$$H = \frac{1}{5}(220 - a)$$

Heart rate, in beats per minute, | is | $\frac{1}{5}$ of | the difference between 220 and your age.

Working It

$$H = \frac{9}{10}(220 - a)$$

Heart rate, in beats per minute, | is | $\frac{9}{10}$ of | the difference between 220 and your age.

The process of finding formulas to describe real-world phenomena is called **mathematical modeling**. Such formulas, together with the meaning assigned to the variables, are called **mathematical models**. We often say that these formulas model, or describe, the relationships among the variables.

EXAMPLE 2 Modeling the Cost of Attending a Public College

The bar graph in **Figure P.1** shows the average cost of tuition and fees for public four-year colleges, adjusted for inflation. The formula

$$T = 4x^2 + 330x + 3310$$

models the average cost of tuition and fees, T, for public U.S. colleges for the school year ending x years after 2000.

a. Use the formula to find the average cost of tuition and fees at public U.S. colleges for the school year ending in 2010.

b. By how much does the formula underestimate or overestimate the actual cost shown in **Figure P.1**?

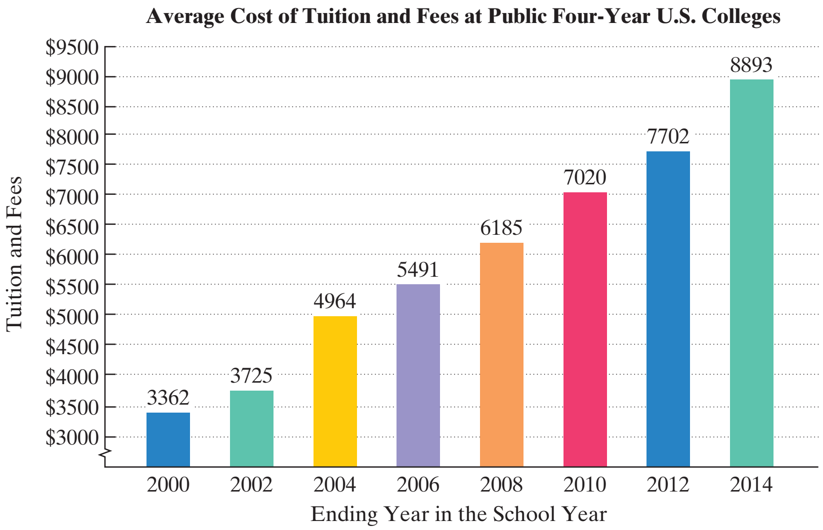


FIGURE P.1
Source: The College Board

SOLUTION

a. Because 2010 is 10 years after 2000, we substitute 10 for x in the given formula. Then we use the order of operations to find T, the average cost of tuition and fees for the school year ending in 2010.

$T = 4x^2 + 330x + 3310$	This is the given mathematical model.
$T = 4(10)^2 + 330(10) + 3310$	Replace each occurrence of x with 10.
$T = 4(100) + 330(10) + 3310$	Evaluate the exponential expression: $10^2 = 10 \cdot 10 = 100$.
$T = 400 + 3300 + 3310$	Multiply from left to right: $4(100) = 400$ and $330(10) = 3300$.
$T = 7010$	Add.

The formula indicates that for the school year ending in 2010, the average cost of tuition and fees at public U.S. colleges was $7010.

b. **Figure P.1** shows that the average cost of tuition and fees for the school year ending in 2010 was $7020.
The cost obtained from the formula, $7010, underestimates the actual data value by $7020 − $7010, or by $10.

Blitzer Bonus || Is College Worthwhile?

"Questions have intensified about whether going to college is worthwhile," says *Education Pays*, released by the College Board Advocacy & Policy Center. "For the typical student, the investment pays off very well over the course of a lifetime, even considering the expense."

Among the findings in *Education Pays*:

- Mean (average) full-time earnings with a bachelor's degree in 2014 were \$62,504, which is \$27,768 more than high school graduates.
- Compared with a high school graduate, a four-year college graduate who enrolled in a public university at age 18 will break even by age 33. The college graduate will have earned enough by then to compensate for being out of the labor force for four years and for borrowing enough to pay tuition and fees, shown in **Figure P.1**.

Check Point 2

a. Use the formula $T = 4x^2 + 330x + 3310$, described in Example 2, to find the average cost of tuition and fees at public U.S. colleges for the school year ending in 2014.

b. By how much does the formula underestimate or overestimate the actual cost shown in **Figure P.1**?

Sometimes a mathematical model gives an estimate that is not a good approximation or is extended to include values of the variable that do not make sense. In these cases, we say that **model breakdown** has occurred. For example, it is not likely that the formula in Example 2 would give a good estimate of tuition and fees in 2050 because it is too far in the future. Thus, model breakdown would occur.

Sets

Before we describe the set of real numbers, let's be sure you are familiar with some basic ideas about sets. A **set** is a collection of objects whose contents can be clearly determined. The objects in a set are called the **elements** of the set. For example, the set of numbers used for counting can be represented by

$$\{1, 2, 3, 4, 5, \ldots\}.$$

The braces, { }, indicate that we are representing a set. This form of representation, called the **roster method**, uses commas to separate the elements of the set. The symbol consisting of three dots after the 5, called an *ellipsis*, indicates that there is no final element and that the listing goes on forever.

A set can also be written in **set-builder notation**. In this notation, the elements of the set are described but not listed. Here is an example:

$$\{x \mid x \text{ is a counting number less than } 6\}.$$

The set of all x | such that | x is a counting number less than 6.

GREAT QUESTION!

Can I use symbols other than braces when writing sets using the roster method?

No. Grouping symbols such as parentheses, ( ), and square brackets, [ ], are not used to represent sets in the roster method. Furthermore, only commas are used to separate the elements of a set. Separators such as colons or semicolons are not used.

The same set written using the roster method is

$$\{1, 2, 3, 4, 5\}.$$

3 Find the intersection of two sets.

If A and B are sets, we can form a new set consisting of all elements that are in both A and B. This set is called the *intersection* of the two sets.

Definition of the Intersection of Sets

The **intersection** of sets A and B, written $A \cap B$, is the set of elements common to both set A **and** set B. This definition can be expressed in set-builder notation as follows:

$$A \cap B = \{x \mid x \text{ is an element of } A \text{ AND } x \text{ is an element of } B\}.$$

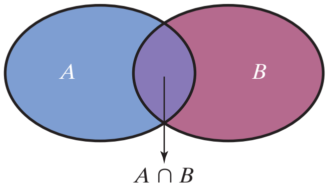


FIGURE P.2 Picturing the intersection of two sets

Figure P.2 shows a useful way of picturing the intersection of sets A and B. The figure indicates that $A \cap B$ contains those elements that belong to both A and B at the same time.

EXAMPLE 3 Finding the Intersection of Two Sets

Find the intersection: $\{7, 8, 9, 10, 11\} \cap \{6, 8, 10, 12\}$.

SOLUTION

The elements common to $\{7, 8, 9, 10, 11\}$ and $\{6, 8, 10, 12\}$ are 8 and 10. Thus,

$$\{7, 8, 9, 10, 11\} \cap \{6, 8, 10, 12\} = \{8, 10\}.$$

Check Point 3 Find the intersection: $\{3, 4, 5, 6, 7\} \cap \{3, 7, 8, 9\}$.

If a set has no elements, it is called the **empty set**, or the **null set**, and is represented by the symbol $\varnothing$ (the Greek letter phi). Here is an example that shows how the empty set can result when finding the intersection of two sets:

$$\{2, 4, 6\} \cap \{3, 5, 7\} = \varnothing.$$

These sets have no common elements.

Their intersection has no elements and is the empty set.

4 Find the union of two sets.

Another set that we can form from sets A and B consists of elements that are in A or B or in both sets. This set is called the *union* of the two sets.

Definition of the Union of Sets

The **union** of sets A and B, written $A \cup B$, is the set of elements that are members of set A **or** of set B or of both sets. This definition can be expressed in set-builder notation as follows:

$$A \cup B = \{x \mid x \text{ is an element of } A \text{ OR } x \text{ is an element of } B\}.$$

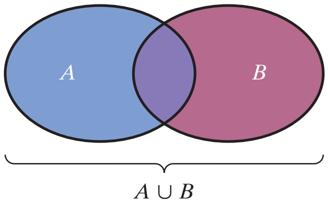


FIGURE P.3 Picturing the union of two sets

Figure P.3 shows a useful way of picturing the union of sets A and B. The figure indicates that $A \cup B$ is formed by joining the sets together.

We can find the union of set A and set B by listing the elements of set A. Then we include any elements of set B that have not already been listed. Enclose all elements that are listed with braces. This shows that the union of two sets is also a set.

EXAMPLE 4 Finding the Union of Two Sets

Find the union: $\{7, 8, 9, 10, 11\} \cup \{6, 8, 10, 12\}$.

SOLUTION

To find $\{7, 8, 9, 10, 11\} \cup \{6, 8, 10, 12\}$, start by listing all the elements from the first set, namely, 7, 8, 9, 10, and 11. Now list all the elements from the second set that are not in the first set, namely, 6 and 12. The union is the set consisting of all these elements. Thus,

$$\{7, 8, 9, 10, 11\} \cup \{6, 8, 10, 12\} = \{6, 7, 8, 9, 10, 11, 12\}.$$

Although 8 and 10 appear in both sets, do not list 8 and 10 twice.

Check Point 4 Find the union: $\{3, 4, 5, 6, 7\} \cup \{3, 7, 8, 9\}$.

GREAT QUESTION!

How can I use the words *union* and *intersection* to help me distinguish between these two operations?

Union, as in a marriage union, suggests joining things, or uniting them. Intersection, as in the intersection of two crossing streets, brings to mind the area common to both, suggesting things that overlap.

5 Recognize subsets of the real numbers.

The Set of Real Numbers

The sets that make up the real numbers are summarized in **Table P.1**. We refer to these sets as **subsets** of the real numbers, meaning that all elements in each subset are also elements in the set of real numbers.

Table P.1 Important Subsets of the Real Numbers

Name/Symbol	Description	Examples
Natural numbers $\mathbb{N}$	$\{1, 2, 3, 4, 5, \ldots\}$ These are the numbers that we use for counting.	$2, 3, 5, 17$
Whole numbers $\mathbb{W}$	$\{0, 1, 2, 3, 4, 5, \ldots\}$ The set of whole numbers includes 0 and the natural numbers.	$0, 2, 3, 5, 17$
Integers $\mathbb{Z}$	$\{\ldots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \ldots\}$ The set of integers includes the negatives of the natural numbers and the whole numbers.	$-17, -5, -3, -2, 0, 2, 3, 5, 17$
Rational numbers $\mathbb{Q}$	$\left\{\frac{a}{b} \,\middle\vert\, a \text{ and } b \text{ are integers and } b \neq 0\right\}$ This means that b is not equal to zero. The set of rational numbers is the set of all numbers that can be expressed as a quotient of two integers, with the denominator not 0. Rational numbers can be expressed as terminating or repeating decimals.	$-17 = \frac{-17}{1}, -5 = \frac{-5}{1}, -3, -2,$ $0, 2, 3, 5, 17,$ $\frac{2}{5} = 0.4,$ $\frac{-2}{3} = -0.6666\ldots = -0.\overline{6}$
Irrational numbers $\mathbb{I}$	The set of irrational numbers is the set of all numbers whose decimal representations are neither terminating nor repeating. Irrational numbers cannot be expressed as a quotient of integers.	$\sqrt{2} \approx 1.414214$ $-\sqrt{3} \approx -1.73205$ $\pi \approx 3.142$ $-\frac{\pi}{2} \approx -1.571$

Notice the use of the symbol $\approx$ in the examples of irrational numbers. The symbol means "is approximately equal to." Thus,

$$\sqrt{2} \approx 1.414214.$$

We can verify that this is only an approximation by multiplying 1.414214 by itself. The product is very close to, but not exactly, 2:

$$1.414214 \times 1.414214 = 2.000001237796.$$

Not all square roots are irrational. For example, $\sqrt{25} = 5$ because $5^2 = 5 \cdot 5 = 25$. Thus, $\sqrt{25}$ is a natural number, a whole number, an integer, and a rational number $\left(\sqrt{25} = \frac{5}{1}\right)$.

The set of *real numbers* is formed by taking the union of the sets of rational numbers and irrational numbers. Thus, every real number is either rational or irrational, as shown in **Figure P.4**.

TECHNOLOGY

A calculator with a square root key gives a decimal approximation for $\sqrt{2}$, not the exact value.

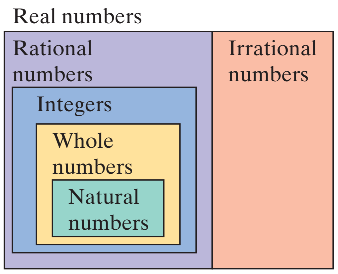


FIGURE P.4 Every real number is either rational or irrational.

Real Numbers

The set of **real numbers** is the set of numbers that are either rational or irrational:

$$\{x \mid x \text{ is rational or } x \text{ is irrational}\}.$$

The symbol $\mathbb{R}$ is used to represent the set of real numbers. Thus,

$$\mathbb{R} = \{x \mid x \text{ is rational}\} \cup \{x \mid x \text{ is irrational}\}.$$

EXAMPLE 5 Recognizing Subsets of the Real Numbers

Consider the following set of numbers:

$$\left\{-7, -\frac{3}{4}, 0, 0.\overline{6}, \sqrt{5}, \pi, 7.3, \sqrt{81}\right\}.$$

List the numbers in the set that are

a. natural numbers. **b.** whole numbers. **c.** integers.
d. rational numbers. **e.** irrational numbers. **f.** real numbers.

SOLUTION

a. Natural numbers: The natural numbers are the numbers used for counting. The only natural number in the set $\{-7, -\frac{3}{4}, 0, 0.\overline{6}, \sqrt{5}, \pi, 7.3, \sqrt{81}\}$ is $\sqrt{81}$ because $\sqrt{81} = 9$. (9 multiplied by itself, or 9^2, is 81.)

b. Whole numbers: The whole numbers consist of the natural numbers and 0. The elements of the set $\{-7, -\frac{3}{4}, 0, 0.\overline{6}, \sqrt{5}, \pi, 7.3, \sqrt{81}\}$ that are whole numbers are 0 and $\sqrt{81}$.

c. Integers: The integers consist of the natural numbers, 0, and the negatives of the natural numbers. The elements of the set $\{-7, -\frac{3}{4}, 0, 0.\overline{6}, \sqrt{5}, \pi, 7.3, \sqrt{81}\}$ that are integers are $\sqrt{81}$, 0, and -7.

d. Rational numbers: All numbers in the set $\{-7, -\frac{3}{4}, 0, 0.\overline{6}, \sqrt{5}, \pi, 7.3, \sqrt{81}\}$ that can be expressed as the quotient of integers are rational numbers. These include $-7\left(-7 = \frac{-7}{1}\right)$, $-\frac{3}{4}$, $0\left(0 = \frac{0}{1}\right)$, and $\sqrt{81}\left(\sqrt{81} = \frac{9}{1}\right)$. Furthermore, all numbers in the set that are terminating or repeating decimals are also rational numbers. These include $0.\overline{6}$ and 7.3.

e. Irrational numbers: The irrational numbers in the set $\{-7, -\frac{3}{4}, 0, 0.\overline{6}, \sqrt{5}, \pi, 7.3, \sqrt{81}\}$ are $\sqrt{5}\left(\sqrt{5} \approx 2.236\right)$ and $\pi(\pi \approx 3.14)$. Both $\sqrt{5}$ and π are only approximately equal to 2.236 and 3.14, respectively. In decimal form, $\sqrt{5}$ and π neither terminate nor have blocks of repeating digits.

f. Real numbers: All the numbers in the given set $\{-7, -\frac{3}{4}, 0, 0.\overline{6}, \sqrt{5}, \pi, 7.3, \sqrt{81}\}$ are real numbers. ● ● ●

Check Point 5 Consider the following set of numbers:

$$\left\{-9, -1.3, 0, 0.\overline{3}, \frac{\pi}{2}, \sqrt{9}, \sqrt{10}\right\}.$$

List the numbers in the set that are

a. natural numbers **b.** whole numbers **c.** integers.
d. rational numbers **e.** irrational numbers **f.** real numbers.

The Real Number Line

The **real number line** is a graph used to represent the set of real numbers. An arbitrary point, called the **origin**, is labeled 0. Select a point to the right of 0 and label it 1. The distance from 0 to 1 is called the **unit distance**. Numbers to the right of the origin are **positive** and numbers to the left of the origin are **negative**. The real number line is shown in **Figure P.5**.

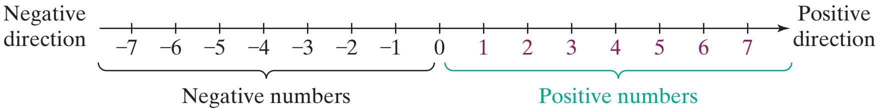
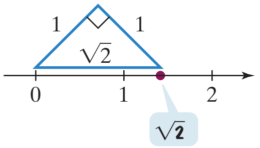


FIGURE P.5 The real number line

GREAT QUESTION!

How did you locate $\sqrt{2}$ as a precise point on the number line in Figure P.6?

We used a right triangle with two legs of length 1. The remaining side has a length measuring $\sqrt{2}$.



We'll have lots more to say about right triangles later in the book.

Real numbers are **graphed** on a number line by placing a dot at the correct location for each number. The integers are easiest to locate. In **Figure P.6**, we've graphed six rational numbers and three irrational numbers on a real number line.

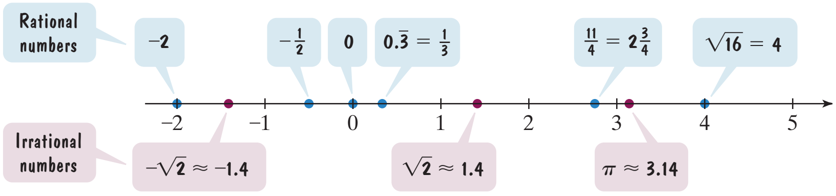


FIGURE P.6 Graphing numbers on a real number line

Every real number corresponds to a point on the number line and every point on the number line corresponds to a real number. We say that there is a **one-to-one correspondence** between all the real numbers and all points on a real number line.

6 Use inequality symbols.

Ordering the Real Numbers

On the real number line, the real numbers increase from left to right. The lesser of two real numbers is the one farther to the left on a number line. The greater of two real numbers is the one farther to the right on a number line.

Look at the number line in **Figure P.7**. The integers -4 and -1 are graphed.

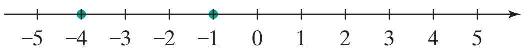


FIGURE P.7

Observe that -4 is to the left of -1 on the number line. This means that -4 is less than -1.

$$-4 < -1$$

-4 is less than -1 because -4 is to the left of -1 on the number line.

In **Figure P.7**, we can also observe that -1 is to the right of -4 on the number line. This means that -1 is greater than -4.

$$-1 > -4$$

-1 is greater than -4 because -1 is to the right of -4 on the number line.

The symbols $<$ and $>$ are called **inequality symbols**. These symbols always point to the lesser of the two real numbers when the inequality statement is true.

-4 is less than -1. $\quad -4 < -1 \quad$ The symbol points to -4, the lesser number.

-1 is greater than -4. $\quad -1 > -4 \quad$ The symbol still points to -4, the lesser number.

The symbols $<$ and $>$ may be combined with an equal sign, as shown in the following table:

Symbols	Meaning	Examples	Explanation
$a \leq b$	a is less than or equal to b.	$2 \leq 9$ $9 \leq 9$	Because $2 < 9$ Because $9 = 9$
$b \geq a$	b is greater than or equal to a.	$9 \geq 2$ $2 \geq 2$	Because $9 > 2$ Because $2 = 2$

This inequality is true if either the $<$ part or the $=$ part is true.

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